# Use of the Niyama criterion to predict porosity of the mushy zone with deformation

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## Abstract

The article presents new results on the use of the Niyama criterion to estimate porosity appearance in castings under hindered shrinkage. The effect of deformation of the mushy zone on filtration is shown. A new form of the Niyama criterion accounting for the hindered shrinkage and the range of deformation localization has been obtained. The results of this study are illustrated by the example of the Niyama criterion calculated for Al-Cu alloys under different diffusion conditions of solidification and rate of deformation in the mushy zone. Derived equations can be used in a mathematical model of the casting solidification as well as for interpretation of the simulation results of casting solidification under hindered shrinkage. The present study results in a new procedure of using the Niyama criterion under mushy zone deformation.

**Keywords:** Solidification, Deformations over the mushy zone, Niyama criterion, Filtration condition, Hindered shrinkage,

## 1. Introduction

The Niyama criterion has been widely used to estimate porosity appearance in castings [1-5]:

$$Niy = \frac{|G|}{\sqrt{|\dot{T}|}} \le [Niy]_{kr} \quad (1)$$

where $\dot{T}$ - cooling rate and $G$ - temperature gradient in the mushy zone in the vicinity of the solidus temperature, $[Niy]_{kr}$ - Niyama criterion critical value. The importance of the Niyama criterion lies in its connection with the filtration condition of the interdendritic liquid in the mushy zone. At a constant value of the Niyama criterion the mushy zone pressure drop providing the liquid deficiency as a result of shrinkage should remain constant. The Niyama criterion critical value corresponds to the critical pressure drop, such that originates shrinkage porosity development.

In the works dedicated to theoretical vindication of the Niyama criterion [1-5], the pressure drop is usually connected with filtration conditions in the mushy zone without regard for its deformation, whereas the critical value of the Niyama criterion is experimentally obtained for specimens of sufficiently simple shape (usually slabs and bars) wherein the hindered shrinkage is not prominent and the mushy zone deformation is low [1-5]. The authors [6], however, have shown that the mushy zone deformation can have a considerable effect on the pressure drop and on the interdendritic liquid filtration process, accordingly. This fact should naturally change the porosity formation conditions. In commercial alloy castings the mushy zone is practically always submitted to deformation because of the mold and core resistance to shrinkage (Fig. 1), and also because of heterogeneous solidification of the casting layers. Therefore, to use the Niyama criterion for intricate profiled surface castings without regard for deformation can introduce considerable errors into porosity prediction. It is thus expedient to account for deformation in the mushy zone using the Niyama criterion to predict porosity in a casting during its solidification in a mold under conditions which are closer to realistic ones.

For this purpose a mathematical model of the interdendritic liquid filtration under deformation of the mushy zone is discussed in the present paper. The model is based on the idea of the casting solidification according to the dendritic mechanism with the development of deformations perpendicular to primary dendrite arms [6] (Fig. 1).

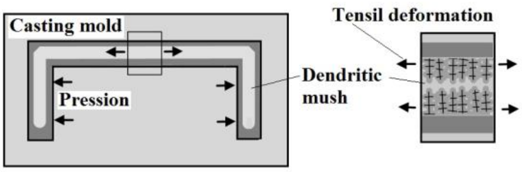


Fig. 1. Appearance of tensile deformation in the casting mushy zone under hindered shrinkage

In the present paper it has also been taken into account that the mushy zone deformations change hydraulic permeability of the dendritic network. The model enabling this phenomenon to be accounted for by means of a varying fraction of solid in a porous medium of the dendrite skeleton is suggested. Another special feature of the present work is the abandonment of the solid fraction linearization across the mushy zone width being usually used for the Niyama criterion derivation [1]. The solid fraction value is being calculated in the present paper from the diffusion conditions in solid and liquid phases.

# 2. Model concept

## 2.1. Change of the solid fraction under deformation

Mushy zone deformations in the direction perpendicular to primary dendrite arms change the primary dendrite arm spacing. An easy calculation scheme can be applied to calculate solid fraction variations as a function of the average primary dendrite arm spacing. Let us designate by $n$ the average number of dendrites per unit length in the mushy zone section perpendicular to primary dendrite arms (Fig. 2a). We shall designate the average cross section area of one dendrite in the perpendicular direction to its primary arm by $s$ , and the initial average primary dendrite spacing in a strain-free state - $\lambda_1$. Then a small volume element of a solid fraction (Fig. 2a) with a perpendicular section to primary dendrite arms $\Delta S = s(n \cdot n)$ and a length $\Delta x$ is equal to $\Delta V_s = \Delta S \cdot \Delta x = s(n \cdot n)\Delta x$ ; the whole volume, including the interdendritic liquid, will be $\Delta V_{s+l} = (n \cdot \lambda_1)^2 \Delta x$. With a certain approximation a relative solid fraction can be written as $f_{s0} = \frac{\Delta V_s}{\Delta V_{s+l}} = \frac{s \cdot (n \cdot n)\Delta x}{(n \cdot \lambda_1)^2 \Delta x} = \frac{s}{\lambda_1^2}$.

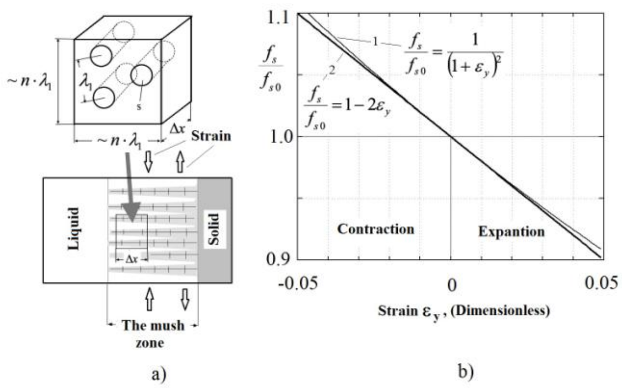


Fig. 2. Deformation effect on solid fraction variation in the mushy zone. a) Schematic diagram of a small volume element with a perpendicular section to primary dendrite arms in the mushy zone, b) Solid fraction variation as a function of relative deformation computed by a more (1) and less (2) exact formulas

Using the obtained formulas one gets:

Changing the average primary dendrite spacing by $\Delta\lambda_1$ it is possible to write:

$$\frac{\Delta f_s}{f_{s0}} = \frac{f_s(\lambda_1 + \Delta\lambda_1) - f_s(\lambda_1)}{f_s(\lambda_1)} = -\frac{2\varepsilon_y + \varepsilon_y^2}{(1+\varepsilon_y)^2}$$

where $f_{s0} = f_s(\lambda_1)$ - relative solid fraction before the deformation appearance, $\varepsilon_y = \Delta\lambda_1 / \lambda_1$ - relative deformation. For small values $\varepsilon_y << 1$ a following approximation is valid:

$$\frac{\Delta f_s}{f_{s0}} \approx -2\varepsilon_y \quad (2)$$

$$f_s = f_{s0} + \Delta f_s = \frac{f_{s0}}{(1+\varepsilon_y)^2} \approx f_{s0}(1 - 2\varepsilon_y) \quad (3)$$

The solid fraction decreases with positive (tensile) deformations and increases with negative (contractive) ones (Fig. 2 b).

## 2.2. Hydraulic permeability

To describe filtration rate $\langle v_x \rangle_l$ of liquid in the interdendritic space of the mushy zone along primary dendrite arms (Fig. 3) one usually makes use of the Darcy equation with a varying permeability coefficient $K = K(f_s, \lambda_2)$ as follows [7,11]:

$$\langle v_x \rangle_l = -\left( \frac{K(f_s, \lambda_2)}{f_l \mu} \right)_x \left( \frac{dp}{dx} + g\rho_l \right) \quad (4)$$

where $p, \mu$ - pressure and dynamic viscosity of the interdendritic liquid, respectively, $\lambda_2$ - average secondary dendrite arm spacing, $g$ – gravitational acceleration.

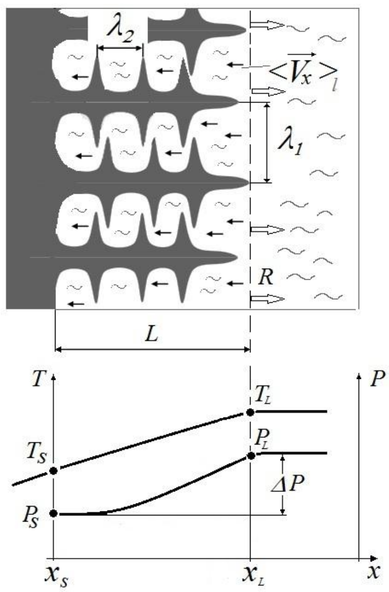


Fig. 3. Filtration model and schematic diagram of temperature and pressure fields over the mushy zone width

The most popular approximation of the permeability coefficient at present is the Kozeny-Carman equation [12-14]:

$$K = \frac{\lambda_2^2}{180\mu}\frac{\left(1-f_s\right)^3}{f_s^2} \quad (5)$$

In view of Eq. (3) for small deformations we get a modified Kozeny-Carman approximation for hydraulic permeability taking dendrite deformation into account:

$$K = \frac{\lambda_2^2\left[1-f_{s0}\left(1-2\varepsilon_y\right)\right]}{180\mu}\left(\frac{1}{f_{s0}\left(1-2\varepsilon_y\right)}-1\right)^2 \quad (6)$$

This equation when there is no deformation coincides completely with its conventional form (5).

Fig.4 shows tabulated values of the permeability coefficient according to Eq. (6), the permeability coefficient rising with tensile deformation ($\varepsilon_y > 0$) and descending with contractive deformation ($\varepsilon_y < 0$).

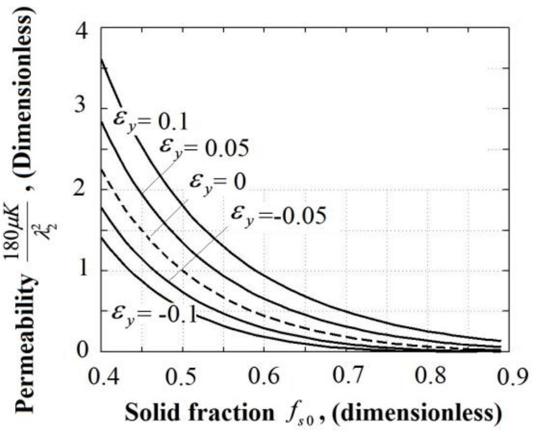


Fig. 4. Deformation effect on the permeability coefficient in the mushy zone. Hydraulic permeability versus solid fraction in the absence of deformation is shown with a dashed curve.

## 2.3. Filtration

For one-dimensional filtration along primary dendrite arms inside a steady-state mushy zone the equation of mass balance can be written according to [6] as follows:

$$\frac{d}{dx}\left(f_l\langle v_x\rangle_l\right)+\left(1+\beta\right)f_s\dot{\varepsilon}_y - R\beta\frac{df_s}{dx}=0 \quad (7)$$

where $R$ - constant isotherm velocity (Fig. 3).

This equation in view of Eq. (4), with account of Eqs. (3) and (6) but ignoring gravity forces is:

$$-\frac{d}{dx}K\left(f_{s0},\varepsilon_y\right)\frac{dP}{dx}+\left(1+\beta\right)f_{s0}\left(1-2\varepsilon_y\right)\dot{\varepsilon}_y - R\beta\frac{df_{s0}}{dx}=0 \quad (8)$$

The last equation can be simplified by the following assumptions: 1) Temperature is linear in the mushy zone (Fig.3.), *i.e.* $\theta = \left(T_L - T\right)/\left(T_L - T_S\right) = x/L$ and 2) deformations are evenly distributed over the whole width of the mushy zone. The first term of Eq. (8) can be approximately expressed via the average filtration coefficient $\tilde{K}$ and the pressure drop as

$$\frac{d}{dx}K\left(f_{s0},\varepsilon_y\right)\frac{dP}{dx} \approx \tilde{K}\frac{P_L-P_S}{L^2}=\tilde{K}\frac{\Delta P}{L^2}$$

where $\Delta P = P_L - P_S$ - pressure drop in the mushy zone (Fig. 3).

Taking into account that

$$\frac{df_{s0}}{dx}=\frac{df_{s0}}{d\theta}\frac{d\theta}{dx}=\frac{df_{s0}}{d\theta}\frac{1}{L}, \quad (9)$$

we get what follows:

$$-\frac{\Delta P}{L^2}\tilde{K}+\left(1+\beta\right)f_{s0}^*\left(1-2\varepsilon_y\right)\dot{\varepsilon}_y+\frac{df_{s0}^*}{d\theta}\frac{R\beta}{L}=0 \quad (10)$$

where $\Delta P$ - pressure drop, $\tilde{K}$ - average permeability coefficient in the mushy zone, $\theta = \left(T_L - T\right)/\left(T_L - T_S\right)$ - relative temperature, $f_{s0}^*$ - solid fraction value for which the Niyama criterion is calculated (this value is usually set at $\theta = 0.9$). It should be noted that the average permeability coefficient depends on the deformations as well: it increases with tensile deformations and decreases – with contractive ones.

As has been mentioned earlier, in the majority of works dedicated to the Niyama criterion, it is assumed that the solid fraction is linearly distributed over the mushy zone width. This assumption is quite acceptable with the deformation effect ignored, as the critical Niyama criterion value is determined experimentally, and therefore the case in question is the scaling of values. When the effect of deformations is taken into account, it is necessary to compare the shrinkage contribution and the deformation contribution to the pressure drop related to a lack of feeding. In this case the solidification rate over relative temperature $df_{s0}/d\theta$ is of considerable importance [see Eq. (10)] and correspondingly a more accurate estimate of the solid phase distribution is required. The solid phase distribution over temperature and the solidification rate can be determined if we use the Brody - Flemings diffusion model of microsegregation [15] with a correction factor obtained from the Clyne - Kurz model [16]. In this case the equation of a relative solid fraction for a binary alloy (with a linear solute concentration dependence of the liquidus temperature) [16] takes the form:

$$f_{s_0} = \left(\frac{1}{1-2\alpha' k}\right)\left\{1-\left(\frac{T_m - T}{T_m - T_L}\right)^{(1-2\alpha' k)/(k-1)}\right\} \quad (11)$$

where $T_m$ - melting point of the pure metal (solvent), $\alpha' = 4D_s t_f / \lambda_2^2$ - diffusion Fourier number, $D_s$ - diffusion coefficient in the solid, $t_f$ - solidification time, $k$ - partition coefficient of solute between liquid and solid phases, for most cast alloys being $k < 1$. When $\alpha' = 0$ there is no back-diffusion[15], and in this case the Brody - Flemings model coincides with the Scheil - Gulliver model[8,9]. The value of $\alpha' = 0.5$ corresponds to the equilibrium lever-rule, *i.e.* thermodynamic equilibrium between liquid and solid phases. Thus, this model includes all diffusion conditions usually observed in the mushy zone. However, the thermodynamic equilibrium is generally associated with infinitely slow cooling ($t_f \to \infty$) at finite diffusion coefficients. This fact should result in infinite dimensionless Fourier numbers $\alpha'$. And so the physical meaning of the diffusion number $\alpha'$ is lost when it tends to 0.5. However, as Clyne and Kurz showed in [10], this situation can be improved if we introduce a factor $\alpha$ varying between zero and infinity. When $\alpha$ tends to infinity, diffusion conditions are approaching the equilibrium ones ($\alpha' \to 0.5$); but $\alpha$ tending to zero ($\alpha' \to 0$) means that the diffusion conditions comply with the Scheil - Gulliver model.

Based on this theory, let us as an example consider filtration flow in the mushy zone for the binary Al-Cu 2%at Cu alloy. Fig. 5 shows the tabulated solid fraction values and solidification rate according to Eq. (11) with the parameters listed in Table 1. As can be seen from Fig. 5a, the solid fraction variation as a function of relative temperature is substantially nonlinear and the dimensionless solidification rate $df^*_{s0}/d\theta$ when $\theta = 0.9$ is much less than unity (Fig.5b). From this it follows that the actual pressure drop in the mushy zone should be much less than that under the assumption of the linear solid fraction distribution ($df^*_{s0}/d\theta = 1$).

**Table 1: List of parameters of Al-Cu alloys (2%at) used in calculation (sourse data [24])**

| Parameter | Value |
|---|---|
| Melting point of pure aluminium $T_m$ (°C) | 660.4 |
| Design coefficient of copper solubility $k$ | 0.145 |
| Liquidus temperature (°C) | 643.3 |

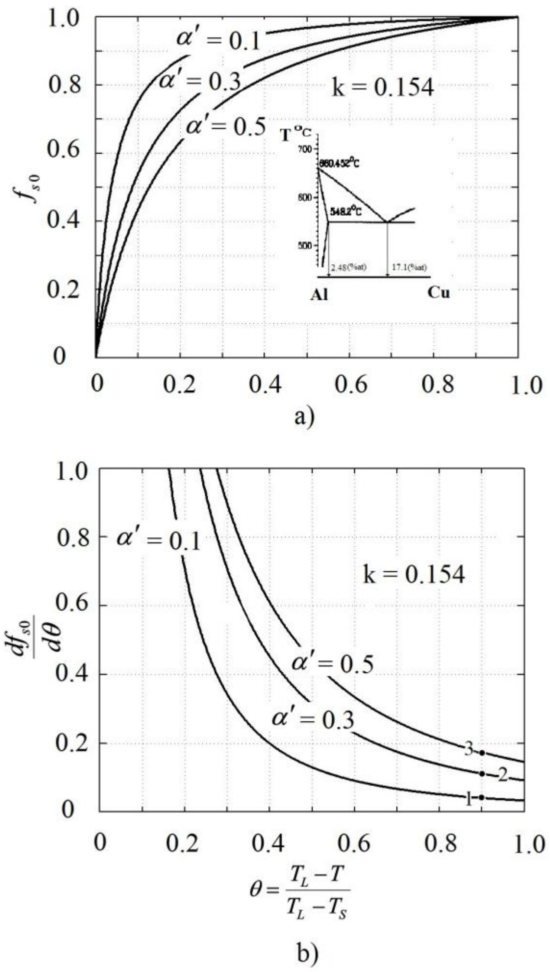


Fig. 5. Calculated values of the relative solid fraction and crystallization rate in a stain-free mushy zone for the Al-Cu alloy: a) solid fraction versus relative temperature at different diffusion numbers, b) dimensionless crystallization rate versus temperature at different diffusion numbers. Points 1, 2, 3 correspond to the crystallization rate at a dimensionless temperature θ set equal to 0.9: $(df_{s0}/d\theta)_1 = 0.0015$, $(df_{s0}/d\theta)_2 = 0.0104$, $(df_{s0}/d\theta)_3 = 0.0246$.

## 2.4. Pressure drop

Accounting for the assumption that the temperature distribution in the mushy zone is close to linear, the average temperature gradient can be written as follows: $G = (T_L - T_S)/L$. The average cooling rate of a point in the mushy zone (solidification time) is $\dot{T} = (T_L - T_S)/t$, where $t = L/R$ - duration of the point stay in the mushy zone (solidification time), therefore average cooling rate will be: $\dot{T} = (T_L - T_S)/t = (T_L - T_S)/(L/R) = R \cdot G$. Hence, taking account of the expression for the Niyama criterion (1), the following relationship is true:

$$|R| \cdot L = (T_L - T_S)/(Niy)^2 \quad (12)$$

Using the relationship (12), Eq. (10) can be thus transformed:

$$\Delta P = \Delta p_\beta + \Delta p_{st} \quad (13)$$

where $\Delta p_\beta = \beta(T_L - T_S)(df^*_{s0}/d\theta)/(Niy)^2 \tilde{K}$ -pressure drop associated with shrinkage processes,

$\Delta p_{st} = (1+\beta) f^*_{s0}(T_L - T_S)^2 (1-2\varepsilon_y)\dot{\varepsilon}_y/(Niy)^2 \tilde{K}|\dot{T}|$ - pressure drop due to lateral deformations of the mushy zone. Note that the second term of Eq. (13) includes both, the deformation rate $\dot{\varepsilon}_y$ and the deformation $\varepsilon_y$ as a multiplier $(1-\varepsilon_y)\dot{\varepsilon}_y$. This circumstance results in stronger effect of the deformation rate on the additional pressure drop at the initial stage of the filtration process with $\varepsilon_y << 1$ than at the later ones.

## 2.5. Effective Niyama criterion

To use the customary procedure for porosity prediction with the Niyama criterion (see Introduction) the obtained results should be brought to the form when all quantities appearing in the Niyama criterion are intact and its critical value changes to fit the deformation conditions of the mushy zone. To do this, let us first transform Eq. (10) as follows:

$$\Delta P = \frac{(T_L - T_S)\beta}{\tilde{K}(Niy)^2} \frac{df^*_{s0}}{d\theta} \left[ 1 + \frac{(1+\beta) f^*_{s0} (T_L - T_S)(1-2\varepsilon_y)\dot{\varepsilon}_y}{\frac{df^*_{s0}}{d\theta} \beta |\dot{T}|} \right]$$

(14)

It is possible hence to derive the effective Niyama criterion value:

$$Niy_{ef} \le n[Niy]_{kr} \quad (15)$$

where $n = \sqrt{1 + (1+\beta) f^*_{s0}(T_L - T_S)(1-2\varepsilon_y)\dot{\varepsilon}_y / |\dot{T}| \beta \, d\xi/d\theta}$ - deformation effect factor (DEF).

The distinction between the expression (15) and the usual form of the Niyama criterion (1) lies in the presence of the multiplier *n* (deformation effect factor, DEF) dependent on alloy properties and deformation conditions in the mushy zone.

# 3. Discussion of results

The results of the present work show that the deformation of the mushy zone is associated with the change of filtration conditions of the interdendritic liquid. The greater the value of the Niyama criterion, the less is the pressure drop required to make up the liquid deficiency and, therefore, the better feeding conditions. This fact corresponds to conventional interpretation of the Niyama criterion. However, Eq. (15) contains quantities that as well affect the pressure drop: the effective or average filtration coefficient $\tilde{K}$ depending on the deformation of dendrites, and the second summand demanding an additional pressure drop to compensate for a lack of feeding associated with expansion of the dendrite skeleton in the perpendicular to the dendrite growth direction. Let us consider approximate estimates of the DEF values. Note that under tensile deformations of constant rate $\dot{\varepsilon}_y = const > 0$ the quantity $(1-2\varepsilon_y)\dot{\varepsilon}_y$ takes the maximum value when $\varepsilon_y = 0$, *i.e.* at the first instant the deformation appeared. Note next that the quantity $(T_L - T_S)/|\dot{T}| = t$ is the time of the mushy zone solidification and the product $(T_L - T_S)\dot{\varepsilon}_y/|\dot{T}| = (\varepsilon_y)_{max}$ is the maximum deformation of the mushy zone at the end of its solidification. Tensile deformations are conveniently estimated in comparison with the linear shrinkage $\alpha$ of the phase transfer from liquid to solid state, which is expressed in terms of the volumetric shrinkage as $\alpha = \beta/3$. Let us introduce a relative quantity

$$\gamma = (\varepsilon_y)_{max}/\alpha \quad (16)$$

and call it a coefficient of deformation localization. When $\gamma = 0$, deformations are missing; when $\gamma = 1$, the shrinkage is completely stopped; and when $\gamma > 1$, increased localization of deformations takes place.

With regard for the latter the maximum DEF (during coupling of dendrites with formation of the bridged skeleton and with $\varepsilon_y = 0$) can be rewritten as follows:

$$n_{max} = \sqrt{1 + f^*_{s0}\gamma(1+\beta)/(3df^*_{s0}/d\theta)} \quad (17)$$

The quantity $n_{max}$ shows the increase in the critical Niyama criterion value required to account for the effect of deformations in the mushy zone. Thus, to prevent the appearance of porosity under tensile deformations, it is necessary to have higher values of the Niyama criterion, and, therefore, higher temperature gradients and lower cooling rates than under conditions without deformations. Fig. 6 shows the tabulated values of this coefficient for alloys with the coefficient of volumetric shrinkage $\beta = 0.06$ (corresponding to steel and some aluminum alloys) at different values of the coefficient of deformation localization and diffusion conditions (diffusion Fourier numbers according to the Brody - Flemings model [15]).

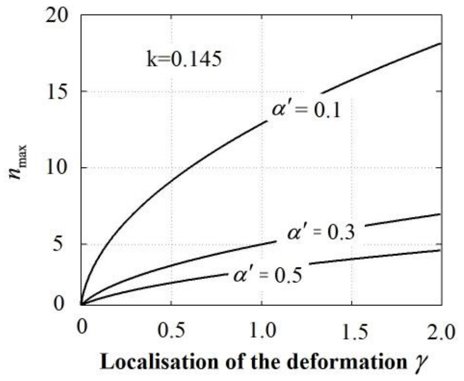


Fig. 6. Maximum deformation effect factor versus coefficient of deformation localization

As it follows from Fig. 6, the effect of the deformation localization on the coefficient $n_{max}$ considerably and to a great extent depends on the diffusion Fourier number $\alpha'$ in the solid phase. Rapid casting cooling ($\alpha'$< 0.1) increases the coefficient $n_{max}$. High values of the coefficient $n_{max}$ are as well retained during relatively slow casting cooling ($\alpha' > 0.3$), even under conditions of thermodynamic equilibrium between liquid and solid phases (this state corresponds to $\alpha' = 0.5$). The obtained results make it possible to use already existing procedures to estimate porosity with the Niyama criterion even in the presence of deformations in the mushy zone. It is necessary for this purpose to adjust the critical value of the Niyama criterion with the help of the coefficient $n_{max}$ which depends on a supposed or calculated coefficient of deformation localization $\gamma$. The $n_{max}$ values shown in Fig. 6 indicate that the critical value of the Niyama criterion can increase several times under tensile deformations. Therefore, resistance to shrinkage which causes tensile deformations is dangerous not only because it leads to hot-tearing formation but it as well increases the hazard of the casting damage with porosity.

# 4. Conclusions

Thus, the analysis of the filtration flow in the interdendritic space of the mushy zone accounting for deformations is given in this work. The influence of deformations on the hydraulic permeability coefficient, heat transfer conditions and pressure drop caused by the filtration flow has been studied. The influence of deformations has been analyzed to suit the diffusion conditions in liquid and solid phases. The results can be presented in the following statements:

1. Deformations in the mushy zone have a strong effect on the pressure drop and, therefore, on the conditions of porosity formation. The critical Niyama criterion value can be several times higher under tensile deformations than in a case when deformations are missing. This fact can explain in particular porosity formation in castings manufactured with rigid molds, whereas computer programs simulating solidification predict its absence.

2. The effect of deformations on porosity formation depends considerably on the diffusion conditions in liquid and solid phases. Low solidification rates favouring the equilibrium solidification conditions are preferable for the production of castings without shrinkage porosity even in the presence of tensile deformations. On the contrary, at high solidification rates the crystallization rate in the vicinity of the solidus temperature decreases what results in the increased deformation contribution to the pressure drop and, therefore, in the hazard of porosity formation under hindered shrinkage, especially with rigid, non-yielding molds.

# 5. References


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